A Distributed Game Theoretic Approach for Optimal Battery Use in an Energy Community

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***Abstract*— With the recent rise of decentralized energy resources like solar panels and batteries, energy communities have grown in popularity. Without collaborative load management, some of the locally produced electricity in the energy community could be unused and sold back to the grid, hindering its economic and environmental benefits. In this paper, we introduce a game to model an energy community with a shared energy source and to determine how to distribute the generated electricity. The goal is to compute a Nash equilibrium so that no participant has an incentive to leave the community. To do so with minimal information sharing between members, a distributed approach is considered. The game structure should incentivize self-consumption in the energy community. We then integrate the game in a receding horizon control framework. This approach is tested on real photovoltaic data and reduces costs for the energy community. Its robustness to imperfect solar forecasting is also evaluated, showing that uncertainty induces minimal increases in cost.**

I. Introduction

Energy communities are a promising way to restructure energy systems to accelerate the energy transition [1]. They have been growing in popularity, especially in Europe, where nearly 4000 citizen-led energy initiatives were recorded in 2023. This has been fostered by the transposition of the EU directives [[2],[3]] into national laws between 2020 and 2022 [4]. An energy community is a localized group of individuals, businesses, or organizations that collaboratively generate, manage, and share distributed energy resources. By combining energy communities with photovoltaic panels and batteries, the amount of energy exchanged with the grid can be significantly reduced, potentially smoothing peaks in the demand during the day [5].

In current energy community, each member often does not have an energy management software; and if they do it does not account for the action of others. Lack of load coordination can lead to sub-optimal community behavior. Having a fully centralized energy community planning also presents issues, especially regarding privacy and fairness within the energy community.

Game theory is a powerful framework that can overcome the shortcomings described above. Game theory can indeed successfully capture the interactions of self optimizing agents with coupled objectives or constraints [6]. Each player has individual objective and a game framework allows one to find a common solution, namely, a Nash equilibrium (NE), from which no player has incentive to deviate. Furthermore, for some specific game, the Nash equilibrium can be computed in a distributed way, reducing privacy concerns.

Some previous works in the literature have applied game theory to energy systems [6], [7], [8], [9], [10], [11], [12], [13], [14], [15], [16]. A popular framework to do so is the mean field game. In such a game, each player do not react to the exact strategies of everyone, but to a statistical aggregation of it. The game can be solved distributively because each player only need to know this shared parameter in addition to their own information. In [7], [8], [9] the shared parameter represents the price of electricity to be purchased from the grid and the Nash equilibrium is found using an iterative algorithm. For it to be successful, the studied energy community are assumed to be very large, so that the mean parameter is statistically significant and the community consumption decision could influence the grid tariff. In [6] and [10], the shared parameter is the mean consumption load of all the players over the horizon. This is used to compute the expected price at each time step. This method is applied to the control of plug-in electric vehicles. It is again assumed that the number of players is large enough to influence the cost of electricity. The example used in both papers controls a fleet of 10^7 electric vehicles.

Another studied game theoretic approach is the Stackelberg game, for which the community manager is decision-maker along with the community members. For example, [11] presents a version of a Stackelberg game where the aggregator has a centralized battery to manage flows between the grid and the consumers. In [12], the leader is a central storage facility aiming at maximising its profits and the followers are the consumers willing to minimize the cost of external energy supply.

In energy systems, the physical constraints can depend on the actions of all the players. In this case, the game becomes a generalized Nash equilibrium problem [17]. An example is [13], which proposes an approach for prosumers community day-ahead planning where the coupling constraint is the energy balance between the different photovoltaic installations. The generalized Nash equilibrium is computed with each player needing the exact strategies of the others.

Several papers combine generalized Nash equilibrium problems with aggregative games when the coupled constraint represents the limited availability of a resource, as it is the case in this work. In [14], the total demand is constrained by the limited capacity of an electrical feeder and a gas pipe. In [15] the total electrical demand is constrained by the grid power limitation and in [16] the coupled constraint is for PV energy consumption. Even if considering setups close to ours, these papers present some difference from our work. For the first two papers, it is assumed that the average consumption level in the community influences the resources per unit cost. However, we consider a community which is

too small to influence the price, so this assumption does not apply to our scenario. The approach in [16] consider some global objective to share between the agent as part of each player objective function, which is not present in our work. Furthermore, they apply their approach to a case where each agent has an individual PV installation, which differs from our scenario where a common central resource has to be shared between the agents.

Overall, these approaches are interesting, but they either are not applicable to small scale energy communities or do not find a solution in a distributed manner. Furthermore, most of them consider day ahead planning rather than real-time. Our objective is to provide a tool for real-time, privacy preserving, consumption load management in energy communities with a shared energy source. This tool should reduce costs for community members by encouraging local energy consumption. To this end, the contributions of this paper are as follows:

- We formulate a game theoretic framework for optimizing battery charge and discharge plans for each members in a small scale energy community. This framework has a clear physical and economical interpretation and allows the community to maximize self-consumption.
- We design an algorithm that can compute a generalized Nash equilibrium for the energy community under consideration in a distributed way, mitigating privacy concerns. This algorithm is also applicable to real-time planning.
- Our extensive simulations show that the proposed approach can significantly reduce costs for energy community members by increasing self-consumption, and that it is robust to imperfect solar forecasts.

II. Problem formulation

TABLE I

NOMENCLATURE

Variables and Cost Functions		
N	-	Number of agents
T	[h]	Time horizon
e_i	[kW]	Electricity consumed by agent i
s	[kW]	Electricity produced by the photovoltaic panels
$p_{lc,i}$	[kW]	Electricity from the panels used by agent i
$p_{buy,i}$	[kW]	Electricity imported from the grid by agent i
p_{sld}	[kW]	Electricity sold to the grid by the community
$p_{ch,i}$	[kW]	Electricity charged in the battery by agent i
$p_{ds,i}$	[kW]	Electricity discharged from the battery by agent i
c_{lc}	$\left[\frac{\text{CHF}}{\text{kWh}}\right]$	Local price for the electricity
c_{buy}	$\left[\frac{\text{CHF}}{\text{kWh}}\right]$	Price for the electricity bought from the grid
c_{sld}	$\left[\frac{\text{CHF}}{\text{kWh}}\right]$	Price for the electricity sold to the grid
SoC_i	[kW]	State of charge of agent i's battery
η_{ch}	-	Charging efficiency of agent i's battery
η_{ds}	-	Discharging efficiency of agent i's battery
β_i	[kWh]	Battery storage capacity
C_{cyc}	$\left[\frac{\text{CHF}}{\text{kWh}^2}\right]$	Degradation cost of agent i's battery
y_i	-	Decision variables of agent i
y	-	Vector of joint actions

We consider an energy community with a set $\mathcal{N} = 1, \ldots, N$ members and a horizon $T \in \mathbb{R}$. Each member i is characterized by an initial demand $e_i \in \mathbb{R}^T$ to be met. To modify their loads, each member possesses a battery energy storage system with a storage capacity $\beta_i \in \mathbb{R}$. Energy communities are often constructed around a community manager whose role is to organize and overview the different physical and economic flows to and from the members of the energy community. The community manager receives the expected solar production $s \in \mathbb{R}^T$ from the central solar installation and dispatches it to the community members. Each member i receives a share of this local resource $p_{lc,i} \in \mathbb{R}^T$, buying it at a cost of $c_{lc} \in \mathbb{R}$. The value of c_{lc} depends on the annualised investment cost and the maintenance costs of the photovoltaic installation. If this energy is not sufficient to meet the members' demand at each time step, they will also consume electricity from the grid, $p_{buy,i} \in \mathbb{R}^T$, at a price $c_{buy} \in \mathbb{R}$. The sum of all these grid imports, $\sum_{i=1}^N p_{buy,i}$, corresponds to the imports of the community $p_{buy} \in \mathbb{R}^T$. If at some point the solar production is greater than the demand, this excess load $p_{sld} \in \mathbb{R}^T$ will be sold back to the grid at a price $c_{sld} \in \mathbb{R}$. The local cost c_{lc} is often designed in a way that is greater than c_{sld} and less than c_{buy}. In this way, consumption within the community is encouraged.

The mutual influence among group members is modeled by diminishing the solar energy available to each individual. At every time steps over the horizon, each agent in the energy community can only consume locally what has not been consumed by the other members. Given the local power consumed by all the other players, $p_{lc,-i} = \sum_{j \neq i} p_{lc,j}$, the local power consumption of player i must satisfy:

$$p_{lc,i} \leq s - p_{lc,-i}, \tag{1}$$

over the entire horizon.

A. Game theoretic formulation of the objectives

The game we introduce is a non-cooperative game with N players. Each player aims at minimizing a cost function depending on their joint decision. At every time step, each player can choose how much to consume locally, and how much to charge and discharge their batteries. For a player i, let the vector $y_i = (p_{ch,i}, p_{ds,i}, p_{lc,i}) \in \mathbb{R}^{3T}$ represents the strategy choice. To satisfy the power balance over the horizon, the following constraint is added:

$$p_{lc,i} + p_{buy,i} = e_i + p_{ch,i} + p_{ds,i}. \tag{2}$$

Each player wants to minimize their billing cost, while accounting for the degradation cost of using the battery. The billing cost contains the cost of using local electricity and the cost of using grid electricity, and it is equal to:

$$J_i^{bill}(y_i) = {p_{lc,i}}^\top c_{lc} + p_{buy,i}^\top c_{buy} \tag{3}$$

Regarding electricity costs, it is here assumed that the energy community is too small to have an impact on the cost of grid electricity c_{buy}. The cost of local electricity c_{lc} is also assumed to be fixed. Thus, both c_{buy} and c_{lc} are set price vectors which are independent of the behavior of energy community members behaviors.

B. Battery model

For each agent i, the following constraints are added to model battery behavior over the entire horizon [7]. Although more complex battery models can be developed, we consider the following constraints as sufficiently precise for the purpose of planning.

- Battery dynamics:

$$SoC_i^{t+1} = SoC_i^t + \frac{\eta_{ch}}{\beta_i} p_{ch,i}^t + \frac{1}{\eta_{ds}\beta_i} p_{ds,i}^t.$$

- State of charge constraints:

$$SoC_{min} \leq SoC_i^t \leq SoC_{max}.$$

- Charge and discharge constraints:

$$p_{ds,max} \leq p_{ds,i}^t \leq 0 \text{ and } 0 \leq p_{ch,i}^t \leq p_{ch,max},$$

where $SoC_i \in [0,1]^T$ is the state of charge of the battery, $p_{ds,i} \in \mathbb{R}^T_{\leq 0}$ is the discharge power of the battery and $p_{ch,i} \in \mathbb{R}^T_{\geq 0}$ the charging power. The charging and discharging efficiencies are, respectively, $\eta_{ch} \in \mathbb{R}$ and $\eta_{ds} \in \mathbb{R}$.

In practice, battery degradation occurs because of several factors, such as the gradual wear and tear of the electrodes due to repeated charging and discharging cycles. Our aim is to minimize this and we thus introduce a cost associated to it. This cost can be defined as a quadratic function of the net power exchanges with the battery [18]. With the diagonal matrix $C_{cyc} \in \mathbb{R}^{T\times T}$ representing the degradation cost per kW used, this cost is:

$$J_i^{batt}(y_i) = \left(\eta_{ch} p_{ch,i} + \frac{p_{ds,i}}{\eta_{ds}}\right)^\top C_{cyc} \left(\eta_{ch} p_{ch,i} + \frac{p_{ds,i}}{\eta_{ds}}\right). \tag{4}$$

C. Individual Players Optimization problem

While the objective function of player i is independent of the actions of the other players, these actions affect the constraints of player i because of (1). This means that the game at hand is a generalized Nash equilibrium problem [17]. This constraint will need to be satisfied for all players when computing their optimal strategies.

Each player i aims to find the strategy y_i^* that minimizes the following optimization problem:

$$\begin{aligned}
\arg\min_{y_i} \quad & J_i^{bill}(p_{lc,i}, p_{buy,i}) + J_i^{batt}(p_{ch,i}, p_{ds,i}) \\
\text{s.t.} \quad & p_{lc,i} + p_{buy,i} = e_i + p_{ch,i} + p_{ds,i} \\
& 0 \leq p_{buy,i}\,, \quad 0 \leq p_{lc,i} \\
& SoC_i^{t+1} = SoC_i^t + \eta_{ch} p_{ch,i}^t + \frac{1}{\eta_{ds}} p_{ds,i}^t \\
& SoC_{min} \leq SoC_i^t \leq SoC_{max} \\
& p_{ds,max} \leq p_{ds,i}^t \leq 0,\ 0 \leq p_{ch,i}^t \leq p_{ch,max} \\
& \underbrace{p_{lc,i} \leq s - p_{lc,-i}}_{\text{Dependence on other players}}\ .
\end{aligned} \tag{5}$$

Let the set $K_i \subset \mathbb{R}^{3T}$ represent the vectors y_i satisfying all the individual constraints, i.e. all but the last constraints in (II-C). To model the influence of agents on each other, the vector of joint actions $y = [y_1 \dots y_N] \in K = K_1 \times \dots \times K_N$ has to belong to the global coupling constraint set $\mathcal{C}$, namely:

$$\mathcal{C} = \{y \in K : g(y) \leq 0\}, \tag{6}$$

with $g : \mathbb{R}^{NT} \to \mathbb{R}^T = (\sum_{i=1}^N p_{lc,i} - s)$ corresponding to the last constraint in (II-C). The optimization problem can be rewritten as:

$$\begin{aligned}
\arg\min_{y_i} \quad & J_i(y_i) \\
\text{s.t.} \quad & y_i \in K_i \\
& g(y) \leq 0.
\end{aligned} \tag{7}$$

The solution set $S_i(y_{-i})$ of the problem (7) depends on the joint actions y_{-i} of the other agents, where $y_{-i} = (y_1, \dots, y_{i-1}, y_{i+1}, \dots, y_N)$. A generalized Nash equilibrium of the game is vector y such that

$$y_i \in S_i(y_{-i}), \quad \forall i \in \{1, \dots, N\}.$$

Due to pratical limits in the computation of a Nash equilibrium, we aim for a solution that approximates it within a small margin of error. This type of solution is called ϵ-Nash equilibrium. An ϵ-Nash equilibrium is a vector y such that

$$J_i(y_i) \leq J_i(\tilde{y}_i) + \epsilon,\ \forall \tilde{y}_i \in \{g(\tilde{y}_i, y_{-i}) \leq 0\} \cup K_i, \forall i \in \{1, \dots, N\}. \tag{8}$$

In the next section, we prove that there exists at least one generalized Nash equilibrium, and we propose an algorithm to compute an ϵ-Nash equilibrium.

III. Analysis and algorithm

Our aim in this section is first to show that the game has at least one generalized Nash equilibrium, and then to design an algorithm that can compute an equilibrium in a distributed way.

A. Existence of a Nash equilibrium

Proposition 3.1: A game with N agents, each one with a cost function as the one described in (7), has at least a generalized Nash equilibrium.

Proof: Notice that the set $\mathcal{C}$, which is defined as the intersection of hyperplanes and half-spaces, is closed and convex. Then, we introduce the map $F : \mathbb{R}^{3NT} \to \mathbb{R}^{3NT}$ which is the pseudo-gradient of the game, defined as $F(y) = [(\nabla_{y_i} J_i(y_i))_{i=1}^N]$. Given that the objective function J_i is quadratic and convex, F is continuous and convex. Theorem 5 in [17] states that, for a so-called *variational inequality* problem, $VI(\mathcal{C}, F)$, the set of solutions is a subset of the generalized Nash equilibrium set of the game. Also, corollary 2.2.5 in [19] states that if $\mathcal{C}$ is compact and convex and F is continuous, then the set of solutions is non-empty. As this is the case with our definitions, there exists a generalized Nash equilibrium for the game. ■

B. Algorithm for computing a Nash equilibrium

We now look at how to design an algorithm that converges to a Nash equilibrium in a distributed way. Doing so would mitigate the data sharing and privacy concerns linked to centralized resolution. The algorithm should converge without each agent knowing the exact strategies of the other agents in the game. The algorithm in [8] also does this to find the Nash equilibrium of a mean field game.

Our approach is inspired by this algorithm, but differs in the shared parameter. Let the parameter $z \in \mathbb{R}^T$ be an estimation of the community load of local energy consumed, namely $\sum_{i=1}^N p_{lc,i}$, and the only information shared with players. Each player would require an individual version of z, z_{-i} which does not account for their own local consumption. The coupling constraint for player i defined in (1) becomes:

$$p_{lc,i} \leq s - z_{-i}. \tag{9}$$

Each player would still solve problem (7), but with this updated coupling constraint.

The optimal strategies could be computed for all players through a two-step iterative algorithm. Given a current iteration step k, each player would first compute their optimal action by solving (7) given $z_{(k)}$, the value of z at iteration k. By collecting each agents optimal $p^*_{lc,i(k)}$, the central coordinator would then compute the resulting estimation of the community local load $\Lambda(z_k)$ as:

$$\Lambda(z_{(k)}) = \sum_{i=1}^{N} p^*_{lc,i(k)}. \tag{10}$$

Notice that the central coordinator does not perform any optimization and does not control any parameters. Rather, it sums the loads of all agents and communicates this aggregate value to everyone, without disclosing the private consumption information. If privacy is not an issue, the agents could directly share among themselves their consumption without the need of the central coordinator. This would then be used to update the value of z for the next iteration:

$$z_{(k+1)} = (1-\eta)z_{(k)} + \eta\Lambda(z_{(k)}), \tag{11}$$

where $\eta \in (0, 1)$ is the learning rate. These iterations would continue until the updates of $z_{(k)}$ become smaller than a convergence criterion ϵ_{stop}.

Algorithm 1 summarizes the steps of the approach described above. It is similar to the Jacobi-type algorithm presented in [17], but applied in a distributed way. Some other approaches linked with variational inequality theory also exist, such as projected gradient descent. However, to the best of our knowledge, no algorithm has been proved to converge to a Nash equilibrium in this setting [17]. We chose the following algorithm due to its simplicity in distributed implementation, its potential for ensuring privacy guarantees, and its ability to provide a feasible solution at each iteration.

Although we did not prove global convergence of the algorithm, it is clear that if the algorithm converges, it converges to a generalized Nash equilibrium. Looking at the convergence criteria, it can indeed be seen that the algorithm converges if no players change their strategies anymore, which corresponds to a Nash equilibrium.

Algorithm 1 Distributed Jacobi-type algorithm

initialize:
  $z_{(1)} \leftarrow min(\sum_{i=1}^N e_i, s)$, $z_{(1)} \in \mathbb{R}^T$
  $p_{lc,i,(0)} \leftarrow min(e_i, s/N)$ for $i = 1...N$,
  $p_{lc,i,(0)} \in \mathbb{R}^T$
  $k \leftarrow 1$
repeat
  for $i \in N$ **do**
    $z_{-i,(k)} \leftarrow z_{(k)} - p^*_{lc,i,(k-1)}$
    Solve local optimization problem to compute $(y^*_{i,(k)})$
  end for
  $z_{(k+1)} \leftarrow (1-\eta)z_{(k)} + \eta(\sum_{i=1}^N p^*_{lc,i,(k)})$
  $k \leftarrow k + 1$
until $\sum_{i=1}^N ||p^*_{lc,i(k+1)} - p^*_{lc,i(k)}|| \leq \epsilon_{stop}$

IV. Simulation Studies

This section presents several simulation results evaluating the applicability of the proposed approach given real photovoltaic data. The objective is to gain qualitative and quantitative insights on the benefits and robustness of our approach for energy communities.

We first analyzed the outcome of the algorithm over a single planning horizon of 48h. We found that all the agents were able to coordinate to maximize local energy consumption, without having to share their exact consumption plan.

The above behavior corresponds to a day-ahead planning approach, where the forecasts are fixed over the entire horizon. However, PV forecasts become increasingly uncertain with time, so the predictions at the end of the horizon can be significantly off. To mitigate this impact, our approach is implemented in a model predictive control framework with receding horizon. To do so, Algorithm 1 is run at each time step. Every time, it generates an entire optimized battery use sequence, but only the value at the first timestep is used to update the state of charge of the batteries. Before advancing to the next step, the forecast is updated. This is simulated using the python library NRGMaestro™ designed by the Centre Suisse d'Electronique et de Microtechnique [20].

Every time Algorithm 1 was used, convergence was achieved, and so we can conclude that players always reached an ϵ-Nash equilibrium (8).

This section analyzes the performance of receding horizon controllers using the load game approach for energy community management. The energy community studied is composed of a population of size $N = 10$, each member possessing its own battery. Each member's baseload is constructed from a common baseload $e_{base} \in \mathbb{R}^T$ representing a standard residential load with morning and evening consumption peaks. The differences between members are then induced by adding some normal noise to e_{base}. The noise is sampled from a normal distribution with a mean of 0 and a standard deviation of 0.2.

The following parameters were selected:

Battery parameters:

- Storage capacity: $\beta_i = 15$ kWh
- Charging and discharging efficiency: $\eta_{ch} = \eta_{ds} = 95\%$
- Maximum charge and discharge rate: $p_{ch,max} = p_{ds,max} = 7.5$ kW
- Starting SoC: 7.5 kWh (50%)

Economical parameters:

- Grid electricity: 0.2 CHF/kWh
- Local electricity: 0.1 CHF/kWh
- Feed-in tariff: 0.05 CHF/kWh

Community parameters:

- Number of members: $N = 10$

The economical parameters have been selected based on the study in [21]. The battery capacity has been arbitrarily chosen to match the scale of each agent's consumption. All the other battery parameters have been kept at the default values of the NRGMaestro™ tool [20].

A. Real photovoltaic data

The load game approach has been applied to real photovoltaic data from an installation close to Zurich. The dataset used contains forecasted and real solar power time series for several years and locations, with quarter hourly granularity. One installation is selected from the dataset, and two days of data are sampled from February 2023.

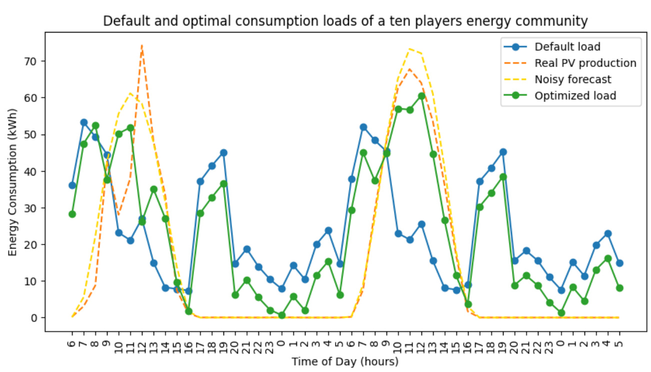

Fig. 1. Default and optimal community loads

A plot showing expected and real solar production, community default consumption load and community optimised load over 48h, starting at 6am. The optimised load follows more the solar load.

Figure 1 shows the initial community load ($\sum_{i=1}^{N} e_i$) and the community load at the generalized Nash equilibrium ($\sum_{i=1}^{N} p_{lc,i} + p_{buy,i}$) and Figure 2 shows the resulting SoC profiles for all the energy community members. Even if the optimizations are only done at the individual level, the load game control approach seems to improve the self-consumption within the entire community. Indeed, the morning and evening peaks relying on grid electricity have been shaven, and have been replaced by another peak during the day, aligned with photovoltaic production.

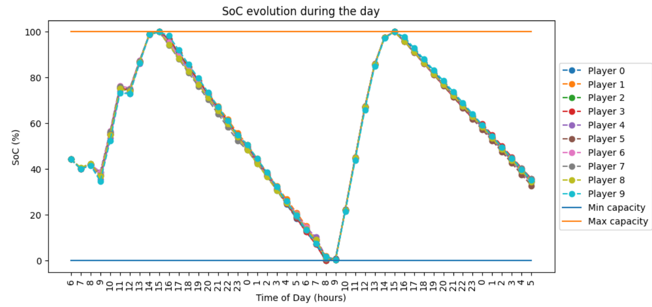

Fig. 2. Optimal SoC profiles

A plot showing the evolution of the battery state of charge for the ten community members over 48h. Each curve discharges until 9am and 40% and then reach full charge at 17pm, before discharging linearly to 0% by 9pm the next day. It then reaches again full charge around 17h, before linearly discharging until the end of the horizon and around 30%.

Looking at the SoC profile, the load game approach successfully pushed energy community members to charge their battery when photovoltaic production is high and discharge it when it is low. The coordination enabled by the load game also ensures that the total power consumed for battery charging by the community does not exceed the photovoltaic power available.

As seen in figure 1, even if using an imperfect photovoltaic forecast, the load game still manages to improve self-consumption in the energy community.

B. Sensitivity to photovoltaic power available

Our goal in this section is to evaluate the sensitivity of the load game results to the quantity of solar energy available. This is important to ensure that the load game approach remains useful for a wide range of implementation conditions, for which the solar production could vary significantly. To quantify this, the parameter $\gamma \in \mathbb{R}$ is introduced. It represents the self production rate, which is the share of total demand over a horizon which can be produced locally over the same period. It is defined as:

$$\gamma = \frac{s^\top 1}{x^\top 1}.$$

The higher the γ, the more solar energy is available. Note that γ represents aggregate values of the parameters over the horizon, but does not assess their time granularity. Having $\gamma = 1$ does not necessarily mean that no electricity is imported from the grid. Imports depends on when the consumption takes place and on the maximum capacity of the batteries. This analysis is done on a one day control horizon with a simulated typical solar load, with peak production during the day and no production during the night.

Figure 3 shows the community costs for different values of γ. As expected, the more solar energy is available the lower is the cost for the community. This is because a greater share of the demand can be met using the cheaper local resources.

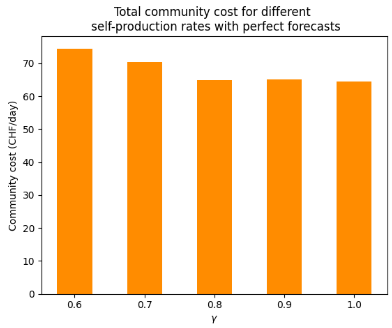

Fig. 3. Community costs for different values of self-production rates

A bar graph representing the energy cost for energy communities for different self production rate, ranging from 0.6 to 1. The cost decreases at first before reaching a plateau after 0.8

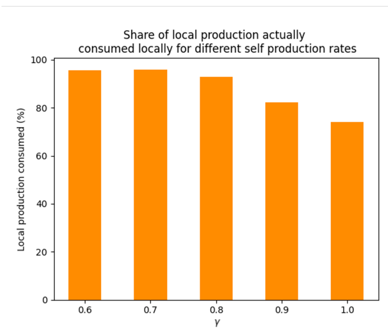

Fig. 4. Local production consumed locally for different values of self-production rates

A bar graph representing the amount of local energy consumed locally for different self production rate, ranging from 0.6 to 1. All energy is consumed before $\gamma = 0.8$, after which it decreases with γ.

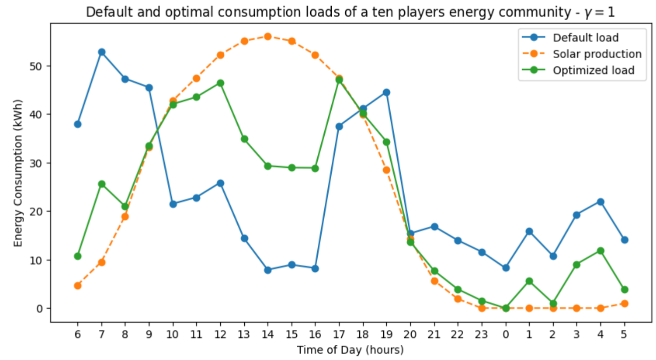

Fig. 5. Default and optimal community loads with $\gamma = 1$

A plot showing solar production, default consumption load and optimised load over 24h, starting at 6am. The optimised load follows more thesolar load.

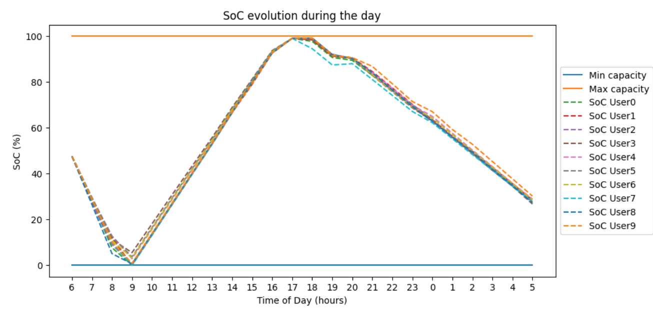

Fig. 6. Optimal SoC profiles with $\gamma = 1$

A plot showing the evolution of the battery state of charge for the ten community members. Each curve reaches full discharge around 9am and full charge at 17pm, before discharging linearly to 20% by the end of the horizon.

However, for γ greater than 0.8, the cost reaches a plateau, after which increasing the amount of local energy does not reduce the cost.

To understand why increasing the local energy available does not decrease cost after a certain point, it is interesting to look at whether all the available solar energy is consumed. Figure 4 shows the share of available local energy consumed locally ($p_{lc,i}{}^{\top}1/s^{\top}1$) for different values of γ. For values of γ where the cost is decreasing, the entirety of the available solar energy is consumed. The amount of solar energy consumed increases, leading to cost savings. However, for $\gamma \geq 0.8$, this percentage gradually decreases as γ increases. A lower percentage of a higher amount of energy is consumed, which makes the absolute amount of solar energy consumed nearly constant. This explains why no additional savings are induced by increasing γ.

The optimal loads and battery use with $\gamma = 1$ are presented in Figures 5 and 6. Not all the photovoltaic energy available is consumed by the community. There is also still some dependence on the grid at the beginning of the horizon and at the end. Looking at the SoC profiles, without using the entire solar energy available, the battery of each player already reaches its maximum capacity. If more photovoltaic energy were consumed earlier on to match the available resources at that time, the maximum capacity would have been reached earlier on, causing the same amount of local resources not to be consumed.

C. Effect of uncertainty

The effect of uncertainty is assessed using the same simulated solar load as in section IV-B. The total community cost is compared for different scenarios ranging from 30% underestimations to 30% overestimation of the real photovoltaic production. This analysis also includes a perfect forecast scenario to use as benchmark. This is done for $\gamma = 0.6$ and $\gamma = 0.8$.

Figure 7 shows the community costs for the different scenarios studied. The cost of importing energy (without accounting for battery use) is higher for all uncertainty scenarios than for perfect forecast.

For underestimation, the battery use cost described in equation (4) gets gradually smaller, but the energy cost described in (3) increases. Because the expected solar production is smaller, the incentive to use the battery is low,

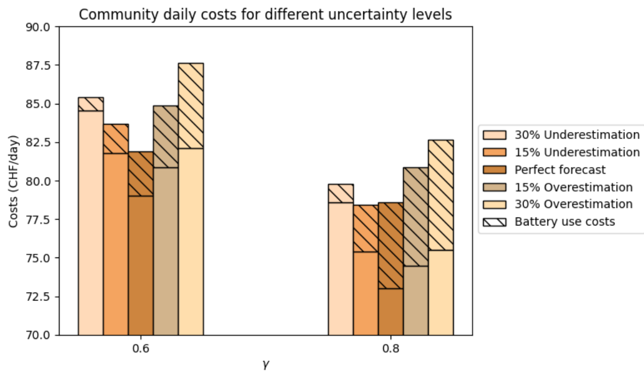

Fig. 7. Impact of imperfect forecasts on community costs for different self-production rates

A bar graph showing how different uncertainty levels influence the cost for the energy community. For $\gamma = 0.6$ and $\gamma = 0.8$, the energy and battery use costs are displayed for uncertainties of 30% underestimation, 15% underestimation, 15% overestimation,30% overestimation and perfect forecasts. Perfect forecast has the lowest cost, then, underestimation errors increases energy costs and reduces battery costs, and overestimation increases battery and energy costs.

resulting in optimized loads being closer to the initial load.

For overestimations, the battery use cost and the energy costs increases. As more photovoltaic production is expected the incentive is now to use the battery more. However, the real solar production is smaller, resulting in the use of imported energy to charge the battery, causing the cost increase.

The above analysis provide interesting insights on the qualitative impact of uncertainty on performance, but when looking quantitatively at the cost increases caused by uncertainty, they seem relatively low. In the worst cases, when uncertainty reaches 30%, the cost does not increase by more than 10%.

Using a load game distributed optimizer seems to ensure that forecasting errors only cause small and acceptable losses in the performance of the controller.

V. CONCLUSIONS

In this paper, we have designed a new game theoretic framework to optimally manage small scale energy communities. The interactions between participants are modeled as a reduction in the available local energy for each player. Such a game can be solved distributively and can be implemented as the optimizer of a receding horizon controller. We have showed that doing so enables the community to maximize self-consumption given the battery capacity at hand. It also allows to generate battery use sequences which are resilient to noisy forecasts.

To build upon the work of this paper, the load game can be applied to more complex energy systems. For instance, it could be applied to systems that integrate both electrical and heating components, or those featuring dynamic energy tariffs, or scenarios where each agent possesses photovoltaic panels instead of relying on a central shared installation. Another possible direction involves stochastic events, such as fluctuations in solar production and agents' energy consumption patterns. Although we investigated the impact of solar production variability in our setting, our algorithm was not optimized for handling such stochastic elements. These represent promising areas for future exploration within the context of the load game framework.

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